APPLICATION OF CAUCHY THEORY IN SOLVING DIFFERENTIAL EQUATIONS IN COMPLEX ANALYSIS

Fina Istifada[1], Anisatul Aeni[2], Dwi Agustina Lestari[3], Umi Mahmudah[4]
[1,2,3,4]*UIN K.H. Abdurrahman Wahid Pekalongan*
finaistifada77@gmail.com
anisatulaeni15@gmail.com
dwiagustinalestari28@gmail.com
umimahmudah@uingusdur.ac.id

ABSTRAK

Fokus pada penelitian ini yaitu untuk menggali implementasi metode *Cauchy* dalam penyelesaian masalah diferensial yang terkait dengan domain kompleks matematika. Tujuan penelitian ini untuk menganalisis penerapan teori *Cauchy* dalam menyelesaiakan persamaan diferensial pada analisis kompleks. Penelitian ini merupakan penelitian kualitatif dengan pendekatan *study literature* melalui studi kasus. Metode pengumpulana data dalam penelitian ini adalah *study literature* dengan mengkaji beberapa literatur; buku, jurnal, dan artikel yang relevan dengan penelitian ini. Penelitian ini menggunakan teknik triangulasi untuk menguji keabsahanan data. Teknik analisis data menggunakan model interaktif meliputi kondensasi data, display data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa teori *Cauchy* dapat diterapkan dalam menyelesaiakan persamaan diferensial pada analisis kompleks. Penelitian ini bermanfaat untuk membantu mahasiswa dalam menyelesaiakan persamaan diferensial pada analisis komplek menggunakan penerapan Teori *Cauchy.*

Kata kunci *:* Teori Cauchy, Persamaan Diferensial, Analisis Kompleks

ABSTRACT

The focus of this research is to explore the implementation of Cauchy's method in solving differential problems related to the complex domain of mathematics. The purpose of this research is to analyze the application of Cauchy's theory in solving differential equations in complex analysis. This research is a qualitative research with a literature study approach through case studies. The data collection method in this research is literature study by reviewing several literatures; books, journals, and articles relevant to this research. This research uses triangulation techniques to test the validity of the data. Data analysis techniques using interactive models include data condensation, data display, and conclusion drawing. The results showed that Cauchy's theory can be applied in solving differential equations in complex analysis. This research is useful to help students in solving differential equations on complex analysis using the application of Cauchy Theory.

Keywords: *Cauchy Theory, Differential Equation, Complex Analysis*

INTRODUCTION

Complex analysis is a branch of mathematics that studies complex functions and the operations that can be performed on them. In complex analysis, differential equations play an important role in explaining natural and technological phenomena. Complex differential equations can be used to describe the behaviour of complex functions associated with technological and natural applications, such as electronic circuits, dynamic systems, and physical phenomena.

However, in some cases, complex differential equations can be very complex and difficult to solve analytically. Therefore, alternative methods are required to solve complex differential equations. One of the methods used is Cauchy theory, which was developed by Augustin-Louis Cauchy in the 19th century. Cauchy theory allows the analysis of complex functions by using the concepts of complex integrals and differentials.

In complex analysis, Cauchy's theory is very useful in solving complex differential equations. Cauchy's theory allows us to describe the behaviour of complex functions using integrals and complex differentials, thus allowing us to solve complex differential equations more easily and effectively. Thus, Cauchy's theory is very important in complex analysis and its applications in technology and nature.

A differential equation is an equation that contains the derivatives of one dependent variable with respect to one or more dependent variables. A differential equation is an equation that contains the derivative of one or more unknown functions where this unknown function is the solution of the given differential equation that must be found. This equation is divided into two namely ordinary differential equations and partial differential equations. An ordinary differential equation is a differential equation that consists of one independent variable and its derivative on that variable. Partial differential equation is a differential equation that consists of more than one independent variable.

Partial differential equation (PDP) is an equation that focuses on the relationship between an unknown function $u(x_1, x_2, \dots, x_n)$ of dimension $n \geq 2$ and the partial derivative of the function with respect to its independent variables (Gunawan, 2021). The general form of PDP is:

$$F\left(\begin{array}{c} x_1, x_2, \dots, x_n, u, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_n}, \frac{\partial^2 u}{\partial x_1 x_1}, \\ \dots, \frac{\partial^2}{\partial x_1 x_n}, \dots \end{array}\right)$$

In the field of complex analysis, the application of Cauchy's theory to differential equations plays an important role. It provides a powerful framework for understanding and solving complex differential equations, which are widely used in various scientific and engineering disciplines (Crawford et al., 1991). Cauchy's theory, developed by mathematician Augustin-Louis Cauchy in the 19th century, establishes a link between complex analysis and differential equations, allowing for the analytical study of functions that satisfy certain differential equations. The theory has many applications in fields such as physics, engineering, and mathematical modelling, where the behaviour of systems can be described by differential equations.

The application of Cauchy theory in solving partial differential equations in complex analysis offers various advantages, including ease of analysis, clear representation of solutions, visualisation capabilities, and effectiveness in high dimensions. This makes Cauchy theory an important tool in various fields of mathematics, physics, and engineering.

Cauchy theory is a fundamental tool in complex analysis that has a close relationship with partial differential equations. Its ability to represent PDP solutions in complex integral form, provide effective complex integral methods, and its effectiveness in high dimensions make Cauchy theory an important tool for solving various PDP problems in various fields of science.

As well as the application of Cauchy theory to the solution of differential equations in previous studies, namely research on the Cauchy-Kovalevsky theorem, the application of Cauchy's theorem in modern analytical methods, the application of semigroup theory in solving Cauchy, and the Cauchy-Euler equation. Some of these studies show how extensive and important the application of Cauchy's theorem is in various aspects of partial differential equations.

Cauchy theory is an important tool in solving partial differential equations. So in this case the researcher is interested in finding the solution of partial differential equations using the application of Cauchy theory.

In this research, researchers will use a qualitative method with a literature study approach or Library Research where data is collected through literature sources or research that is considered relevant to this research. The data collection technique uses documents in the form of previous research that is relevant to this research. Meanwhile, the data analysis uses Miles and Huberman analysis which includes three flows, namely data reduction, data presentation, and conclusion drawing.

DISCUSSION

Differential Equation

Differential equations are mathematical equations for functions consisting of one or more variables where the value of the function will be related to its derivatives in various orders (Edward et al., 2008). Differential equations were first introduced by Newton and Leibniz by making the general form of differential equations, namely:

$\frac{dy}{dx} = f(x)$.........(Krulik et al., 1995)

$\frac{dy}{dx} = f(x, y)$......(Setyawan, 2015) $x_1 \frac{\partial y}{\partial x_1} + x_2 \frac{\partial y}{\partial x_2} = y$......(Smith, 1977)

Then in 1695 Jacob Bernoulli simplified the equation to :

$y' + P(x)y = Q(x)y^n$..........(Edward et al., 2008)

And by Leibniz then found the solution and simplification.

Partial Differential Equation

Partial differential equation is a form of mathematical equation consisting of one or more partial differential operators on the independent variable of a many variable function (Riancelona, 2007). The general form of partial differential equation is:

$$\sum_{i=1}^{N} A_i \frac{\partial^2 f}{\partial x_i^2} + \sum_{i=1}^{N} B_i \frac{\partial f}{\partial x_i} + Cf + D = 0$$

The order of the partial differential equation is the highest derivative that appears in the partial differential equation.

- First order differential equation

  $\frac{\partial C}{\partial x} - a \frac{\partial C}{\partial x} = 0$

- 2nd order differential equation

  $\frac{\partial^2 C}{\partial x^2} - Dc\frac{\partial C}{\partial x} = 0$

- 3rd order differential equation

  $\left(\frac{\partial^3 u}{\partial x^3}\right)^2 - \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} = 0$

Differential equations are divided into 3, namely elliptic, parabolic, and hyperbolic differential equations. For example, given a partial differential equation of order 2 in x space and t time variables.

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial t} + C\frac{\partial^2 u}{\partial t^2} + D\left(x, t, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial t}\right) = 0$$

Where A, B, and C are functions of x and t, while D is a function of u and derivatives of $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial t}$ and x and t. What distinguishes the three classes of partial differential equations is the discriminant value $B^2 - 4AC$ of the equation.

a. A partial differential equation is said to be a **hyperbolic** equation if the value of the discriminant $B^2 - 4AC > 0$
b. A partial differential equation is said to be a **parabolic** equation if the value of the discriminant $B^2 - 4AC = 0$
c. Partial differential equation is said to be an **elliptic** equation if the discriminant value $B^2 - 4AC < 0$

Cauchy Theory Equation

The definition of Cauchy theory equation is a system of partial differential equations that relates analytic functions of two complex variables. In the context of complex analysis, this equation states that a complex function is holomorphic if and only if it satisfies the Cauchy-Riemann condition.

The general form of the Cauchy functional equation:

$$f(x + y) = f(x) + f(y)$$

It was proved by Cauchy in 1821 that the only continuous solution of the functional equation from R to R is of the form $f(x) = k$, x a real number k.

The general form of the Cauchy-Riemann equation for functions $f(z) = u(x, y) + iv(x, y)$ is : $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ dan $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

And can be expressed by $f'^{(z)} = u_x + iv_x = v_y - iu_y$. Where $u(x,y)$ is the real part of the function and $v(x,y)$ is the imaginary part.

Application of Cauchy's Theory in Differential Equation Solving

The application of Cauchy's theory in solving differential equations is one of the important aspects of complex analysis. Cauchy's theory provides a strong framework for understanding the properties of analytic functions and performing complex integral calculations relevant in solving differential equations. Below, I will describe some of the ways in which Cauchy theory is used in solving differential equations:

1. Cauchy-Goursat Theorem

   The Cauchy-Goursat theorem states that if a complex function is analytic in a domain, then the integral across every closed curve in the domain is zero. An important consequence of this theorem is that the integral of an analytic function along a closed trajectory can be converted to an integral along parallel trajectories without changing its value. This allows us to solve the differential equation by finding the integral along a suitable closed trajectory.

   The Cauchy-Goursat theorem is one of the important results in complex analysis which states that if a complex function $f(z)$ is analytic in a closed domain and the curve γ is a closed trajectory in the domain, then the integral $f(z)$ of the complex function is a closed trajectory γ along is zero.

   $$\oint \gamma\, f(z)dz = 0$$

2. Cauchy Integral Formula

   Cauchy Integral Formula provides a way to calculate the value of a complex function inside a domain by using the values of the function on the boundary of the domain. This formula is very useful in solving differential equations as it allows us to convert the differential equation into integral form and use the Cauchy Integral formula to find the solution.

   The Cauchy Integral formula provides a way to calculate the value of a complex function inside a domain using the values of the function on the boundary of that domain. This is very useful in solving differential equations, especially when we need to calculate the solution in integral form.

$$f(a) = \frac{1}{2\pi i} \oint \gamma \frac{f(z)}{z-a} dz$$

3. Residue of Cauchy's Theorem

   The residue of Cauchy's theorem is an important tool in computing closed integrals of functions that have poles within the domain of integration. In the context of solving differential equations, Cauchy's residue theorem is often used to find particular solutions of partial differential equations by using Laplace transform or Fourier transform.

   The residue theorem is a powerful tool in computing closed integrals of functions that have poles in the integration domain. It is very useful in frequency analysis and dynamic system modeling.

   If $f(z)$ has poles in $z = a$, then:

$$\oint \gamma\, f(z)dz = 2\pi i. Res(f, a)$$

   Where $Res(f, a)$ is the residue $f(z)$ in $z = a$.

4. Singularity Solution

   Cauchy's theorem also helps in solving differential equations involving singularities, such as essential singularities, poles, or other singularity points. By understanding the properties of singularities and using Cauchy's theorem, we can find solutions to differential equations around these singularity points.

5. Convergence Analysis

   Cauchy's theorem is also used to analyze the convergence of complex series that arise in solving differential equations. Using the concept of residues and the complex integral theorem, we can evaluate the convergence of the obtained solutions and ensure that they are valid throughout the relevant domain.

By utilizing Cauchy's theory in solving differential equations, we can expand our understanding of the mathematical properties of the solutions found and produce more accurate and efficient solutions in the context of complex analysis.

This research will discuss one type of Partial Differential Equation (PDP), namely the wave equation. The wave equation is a mathematical equation that describes the nature and behavior of waves in various physical contexts, such as sound waves, water waves, or light waves. The concept is based on the basic principle that waves are changes that propagate through a medium.

The basic concept of the wave equation is that there is a relationship between time (t) and position (x) of a wave. The wave equation can be a partial differential equation or an inner equation, depending on the nature of the wave being studied. For example, the wave equation for a Flood wave is of the form:

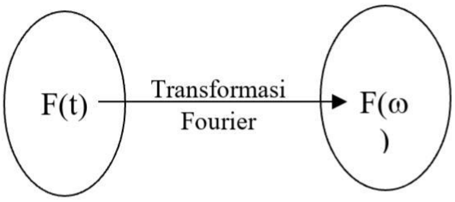
$$\frac{\partial^2 h}{\partial t^2} = g \frac{\partial^2 h}{\partial x^2}$$

In this equation h(x,t) is the height of the wave, g is the acceleration of gravity, x is the horizontal coordinate, and t is time.

The solution of this equation can be done using Cauchy's theory, as follows:

1. Finding the general solution

   Find the general solution of the differential equation using the Fourier method. The Fourier transform or method is a transformation model that transfers the spatial domain or time domain into the frequency domain.



Gambar 4.1. Transformasi Fourier

Fourier transform is a widely used process to move the domain of a function or object into the frequency domain. In digital image processing, the Fourier transform is used to convert the spatial domain of an image into the frequency domain.

In this case, the general solution can be expressed as

$$h(x,t) = H(x)\cos(\omega t + \emptyset)$$

in this equation, H(x) is the horizontal coordinate function, ω is the wave frequency and ∅ is the wave phase.

2. Finding the horizontal coordinate function

   Find the horizontal coordinate H(x) by using the differential equation associated with the flood wave equation. In this case, the horizontal coordinate function can be expressed as

$$H(x) = A\, cos\,(kx)$$

In this equation. A is the amplitude of the wave, and k is the wavenumber.

3. Finding the final solution

   Find the final solution of the flood wave equation by combining the general solution and the horizontal coordinate function. In this case, the final solution can be expressed as:

   $h(x,t) = A\cos(kx)\cos(\omega t + \emptyset)$

   In synthesis, solving the flood wave equation with Cauchy theory involves several steps, including 1, 2, 3. This solution helps in understanding how ocean waves evolve and affect coastal areas.

   Below are some examples of cases that are applications of the flood wave equation using Cauchy's theory:

1. A river has a gravitational acceleration $g = 9{,}8\ m/s^2$. Initially, the water level of the river is expressed by the function $h(x,0) = \cos\left(\frac{\pi x}{4}\right) m$.

   Find the water level $h(x,t)$ at the time $t = 2$ second.

   Solution:

   - Determine the flood wave equation. The flood wave equation based on Cauchy's theory is as follows:

     $$\frac{\partial^2 h}{\partial t^2} = g\frac{\partial^2 h}{\partial x^2}$$

     Where $g = 9{,}8\ m/s^2$

   - Using Fourier Transform

     Fourier Transform of

     $$h(k,t) = \int_{-\infty}^{\infty} h(x,t)e^{-ikx}\,dx$$

     With this, the wave equation in the Fourier domain becomes :

     $$\frac{\partial^2 h}{\partial t^2} = gk^2 h$$

     The general solution to this equation is:

     $$h(k,t) = A(k)\cos(\omega_k t) + B(k)\sin(\omega_k t)$$

     where $\omega_k = \sqrt{g}.k$

   - River initial conditions

     The initial conditions are

     $h(x,0) = \cos\left(\frac{\pi x}{4}\right)$ and $\frac{\partial h}{\partial t}(x,0) = 0.$

The initial condition of $h(x,0)$ in the Fourier Domain is:

$$h(k,t) = \int_{-\infty}^{\infty} \cos\left(\frac{\pi x}{4}\right) e^{-ikx}\, dx$$

- The Fourier transform of

  $\cos\left(\frac{\pi x}{4}\right)$ is

  $$h(k,t) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]$$

  with initial velocity condition

  $\frac{\partial h}{\partial t}(x,0) = 0$, then $B(k) = 0$

- Solution in Fourier Domain

  With $A(k) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]$, the solution in Fourier domain is:

  $$h(k,t) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]\cos(\omega_k t)$$

  with $\omega_k = \sqrt{g}.k$:

  $$h(k,t) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]\cos(\sqrt{9{,}8}.k.t)$$

- Inverse Fourier Transform

  Perform Fourier inversion to get $h(x,t)$:

  $$h(k,t) = \int_{-\infty}^{\infty} h(k,t)e^{-ikx}\, dk$$

  substitution $h(k,t)$:

  $$h(k,t) = \pi\left[cos\left(\frac{\pi x}{4} - \frac{\pi}{4}\sqrt{9{,}8}.t\right) + cos(\frac{\pi x}{4} + \frac{\pi}{4}\sqrt{9{,}8}.t)\right]$$

- Evaluation on $t = 2$

  Substitution $t = 2$:

  $$h(x,2) = \pi\left[cos\left(\frac{\pi x}{4} - \frac{\pi}{4}\sqrt{9{,}8}.2\right) + cos(\frac{\pi x}{4} + \frac{\pi}{4}\sqrt{9{,}8}.2)\right]$$

  $$h(x,2) = \pi\left[cos\left(\frac{\pi x}{4} - \frac{\pi}{2}\sqrt{9{,}8}\right) + cos(\frac{\pi x}{4} + \frac{\pi}{2}\sqrt{9{,}8})\right]$$

- Final solution

  So, the water level at time

  $t = 2$ second is :

  $$h(x,2) = \pi\left[cos\left(\frac{\pi x}{4} - \frac{\pi}{2}\sqrt{9{,}8}\right) + cos(\frac{\pi x}{4} + \frac{\pi}{2}\sqrt{9{,}8})\right]$$

Here is the explicit solution of the water level $h(x,t)$ on time $t = 2$ second.

2. A river has an acceleration of gravity $g = 9{,}8\ m/s^2$. Initially, the water level of the river is expressed by the function

   $h(x,0) = \cos\left(\frac{\pi x}{2}\right) m.$

   Find the water level $h(x,t)$ on time $t = 2$ second.

Solution :

Unknown : acceleration of gravity $g = 9{,}8\ m/s^2$, initial water level function $h(x,0) = \cos\left(\frac{\pi x}{2}\right) m$, Initial velocity $\frac{\partial h}{\partial t}(x,0) = 0$

- Wave equation

  The wave equation we will use :

$$\frac{\partial^2 h}{\partial t^2} = c^2 \frac{\partial^2 h}{\partial x^2}$$

  where $c = \sqrt{gH}$ (wave speed in still water). For simplicity, we assume a fixed value of c.

- Using Fourier Transform

  We will apply the Fourier transform to the wave equation.

  Fourier transform of

  $h(k,t) = \int_{-\infty}^{\infty} h(x,t) e^{-ikx}\, dx$

  With this, the wave equation in the Fourier domain becomes :

$$\frac{\partial^2 h}{\partial t^2} = -c^2 k^2 h$$

  The general solution to this equation is :

$$h(k,t) = A(k)\cos(c_k t) + B(k)\sin(c_k t)$$

- River initial conditions

  Use the initial conditions to determine the coefficients $A(k)$ and $B(k)$. Initial conditional are $h(x,0) = \cos\left(\frac{\pi x}{4}\right)$:

  Intial conditional are $h(x,0)$ in the fourier domain:

$$h(k,t) = \int_{-\infty}^{\infty} \cos\left(\frac{\pi x}{4}\right) e^{-ikx}\, dx$$

  Fourier transform of $\cos(ax)$ is $\pi[\delta(k-a) + \delta(k+a)]$, so that : $h(k,0) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]$, So, $A(k) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]$ with initial velocity condition $\frac{\partial h}{\partial t}(x, 0) = 0$, then $B(k) = 0$

- Solution in the fourier domain

  With $A(k) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]$ dan $B(k) = 0$, Solution in the fourier domain are :

$$h(k,t) = A(k)\cos(ckt)$$

  substitution $A(k)$:

$$h(k,t) = \pi\left[\delta\left(k - \frac{\pi}{4}\right) + \delta(k + \frac{\pi}{4})\right]\cos(ckt)$$

- Inverse Fourier Transform

  Perform Fourier inversion to get $h(x,t)$:

$$h(k,t) = \int_{-\infty}^{\infty} h(k,t)e^{-ikx}\,dk$$

  substitution $h(k,t)$:

$$h(k,t) = \pi\left[\cos\,(c\frac{\pi}{4}t)e^{i\frac{\pi}{4}x} + cos\,(c\frac{\pi}{4}t)e^{-i\frac{\pi}{4}x})\right]$$

  using Euler's identity

  $e^{i\theta} + e^{-i\theta} = 2\cos(\theta)$:

$$h(k,t) = \pi\cos\left(c\frac{\pi}{4}t\right).2\cos\left(c\frac{\pi}{4}t\right)$$

- Evaluation of $t = 2$

  Substitution $t = 2$:

$$h(x,2) = 2\pi\cos\left(9{,}8\frac{\pi}{4}2\right).2\cos\left(x\frac{\pi}{4}\right)$$

  because $c = \sqrt{gH}$, we assume $c = \sqrt{9{,}8}$. For simplicity, so that :

$$h(x,2) = 2\pi\cos\left(\frac{2\pi\sqrt{9{,}8}}{4}\right).2\cos\left(x\frac{\pi}{4}\right)$$

- Final solution

  So, the water level at time $t = 2$ seconds is :

$$h(x,2) = 2\pi\cos\left(\frac{2\pi\sqrt{9{,}8}}{4}\right).2\cos\left(x\frac{\pi}{4}\right)$$

  This is the explicit solution of the water level $h(x,t)$ at time $t = 2$ second.

3. A river has a water level that can be expressed by the function $h(x, 0) = \cos\ (\pi x)\ m$. The river has a gravitational acceleration $g = 9{,}81\ m/s^2$. Find the water level $h(x, t)$ at time $t = 1$ second.

Solution :

To determine the water level

$h(x, t)$ at time $t = 1$ second, we must remember that the water level $h(x, t)$ is likely to change following the wave equation in one dimension with gravitational acceleration $g = 9{,}81\ m/s^2$. The one-dimensional wave equation for water level is usually expressed as follows :

$$\frac{\partial^2 h}{\partial t^2} = c^2 \frac{\partial^2 h}{\partial x^2}$$

Where *c* is the wave speed. For the case of water with gravitational acceleration *g* and depth *h*, the shallow wave speed c can be expressed as follows:

$$c = \sqrt{gH}$$

However, we can simplify it by using the initial height function

$h(x, 0) = \cos \pi x$.

The general solution to the wave equation is:

$$h(x, t) = f(x - ct) + g(x + ct)$$

where *f* and *g* are functions determined by the initial conditions.

Suppose $h(x, 0) = \cos \pi x$, we can choose *f* and *g* so that this solution satisfies the initial condition and the initial velocity condition which is usually

$$\frac{\partial^2 h}{\partial t^2}(x, 0) = 0$$

From the initial condition $h(x, 0) = \cos \pi x$, we can write :

$$f(x) + g(x) = \cos \pi x$$

And for the initial velocity condition $\frac{\partial^2 h}{\partial t^2}(x, 0) = 0$, we can obtain :

$$-cf^I(x) + cg^I(x) = 0$$

This implies that : $f^I(x) = g^I(x)$

So *f* and *g* must have the same form or at least be linearly related.

Suppose we take $f(x) = \frac{1}{2}\cos \pi x$ dan $g(x) = \frac{1}{2}\cos \pi x$, then :

$$h(x, t) = f(x - ct) + g(x + ct) = \frac{1}{2}\cos(\pi(x - ct)) + \frac{1}{2}\cos(\pi(x + ct))$$

with wave speed $c = \sqrt{gH} \approx \sqrt{9{,}81 \times 1} = \sqrt{9{,}8} \approx 3{,}13\ m/s$ at $t = 1$ second :

$$h(x,1) = \frac{1}{2}\cos(\pi(x - 3{,}13 \times 1)) + \frac{1}{2}\cos(\pi(x + 3{,}13 \times 1))$$

$$h(x,1) = \frac{1}{2}\cos(\pi x - 3{,}13\pi) + \frac{1}{2}\cos(\pi x - 3{,}13\pi)$$

using trigonometric identities :

$$\cos(A + B) + \cos(A - B) = 2\cos A \cos B$$

Then

$$h(x,1) = \cos(\pi x) + \cos(3{,}13\pi)$$

From the cosine value, we know that $\cos(3{,}13\pi)$ is a negative value because $3{,}13\pi$ is close to 3π, and $\cos(3\pi) = -1$.

Thus, $\cos(3{,}13\pi) \approx -\cos(0{,}13\pi) \approx -\cos(23{,}4°) \approx -0{,}923$

Therefore :

$$h(x,1) \approx \cos(\pi x) \times (-0{,}923) = -0{,}923\cos(\pi x)$$

So, the water level at $t = 1$ second is $h(x,1) = -0{,}923\cos(\pi x)$ m.

CONCLUSION

From the explanation in the results of the discussion, it can be concluded that Cauchy Theory in the context of wave equations in partial differential equations using the Fourier method can be used to find general solutions of wave equations, especially in flood wave equations. In finding a solution to the flood wave equation using Cauchy theory using three steps, namely 1) finding a general solution where in this step the researcher uses the Fourier method or transformation, 2) then finding a horizontal coordinate function using the differential equation of the flood wave equation, and finally 3) finding the final solution where in this step combining the general solution with the horizontal coordinate function so that the final solution can be found. For further research, it is expected to find the application of Cauchy theory in other types of hyperbolic partial differential equations.

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