# STUDENTS' CRITICAL THINKING PROCESS IN SOLVING COMPLEX NUMBER POLYNOMIAL PROBLEMS BASED ON *FACIONE THEORY*

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#### ABSTRACT

Penelitian ini bertujuan untuk menganalisis proses berpikir kritis siswa dengan kemampuan matematika tinggi, sedang dan rendah dalam menyelesaikan permasalahan bilangan kompleks polinomial berdasarkan teori Facione. Penelitian dilakukan di SMAN 1 Bojong Kabupaten Pekalongan. Penelitian ini menggunakan metode kualitatif melalui analisis deskriptif. Subjek penelitian adalah 5 siswa kelas XI dengan kemampuan matematika tinggi, sedang dan rendah. Instrumen dalam penelitian ini adalah lembar tes dan pedoman wawancara. Penelitian ini menggunakan teknik triangulasi dan sumber keabsahan data. Teknik analisis data yang digunakan adalah kondensasi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa siswa dengan kemampuan matematika lebih tinggi cenderung menggunakan pendekatan yang lebih sistematis dan analitis dalam menyelesaikan masalah, sedangkan siswa dengan kemampuan matematika rendah cenderung mengandalkan trial and error. Proses berpikir kritis juga diamati melalui kemampuan siswa dalam mengidentifikasi pola, memecahkan masalah, dan memberikan justifikasi atas langkahnya. Implikasi pedagogis dari temuan ini menekankan pentingnya mengembangkan keterampilan berpikir kritis melalui pembelajaran terstruktur dan penekanan pada pemahaman konsep matematika dasar.

**Kata Kunci**: Berpikir kritis, polinomial bilangan kompleks, kemampuan matematika, teori Facione

## ABSTRACT

This research aims to analyze the critical thinking process of students with high, medium and low mathematical abilities in solving polynomial complex number problems based on Facione's theory. The research was conducted at SMAN 1 Bojong, Pekalongan Regency. This research uses qualitative methods through descriptive analysis. The research subjects were 5 class XI students with high, medium and low mathematics abilities. The instruments in the research were test sheets and interview guidelines. This research uses triangulation techniques and sources for data validity. The data analysis techniques used are data condensation, data display, and drawing conclusions. The results showed that students with higher mathematical abilities tended to use a more systematic and analytical approach in solving problems, while students with lower mathematical abilities tended to rely on trial and error. The critical thinking process is also observed through students' ability to identify patterns, solve problems, and provide justification for their steps. The pedagogical implications of these findings emphasize the importance of developing critical thinking skills through structured learning and an emphasis on understanding fundamental mathematical concepts.

**Keywords:** Critical thinking, complex number polynomials, mathematical abilities, Facione's theory.

#### **INTRODUCTION**

In the era of globalization, the ability to think critically in the world of education is an important competency for every individual (Pertiwi et al., 2018). Education is a means to develop human thinking. The equality education program standards not only include mandatory material for each level, but there is also skill empowerment material aimed at fostering self-confidence, activeness in activities and appropriateness in determining solutions (Kemendikbudristek, 2022). Education views critical thinking as the stage of obtaining information at the highest level (Van der Zanden et al., 2020). Teachers are responsible for developing ways of thinking by giving problems to students and asking them to criticize and find solutions so that students are encouraged to think deeply and not just provide solutions by relating rote knowledge (Mira Azizah, Joko Sulianto, 2022).

By thinking critically, every individual can adapt to rapid changes in the social sectors of society and the economy (van der Zanden et al., 2020). According to Bloom's taxonomy, there are three top levels in processing information, namely the first is analysis, which is the collection of ideas and concepts that are the basis for determining solutions. The second is evaluation which makes an assessment of the evidence obtained so that it can re-examine the conclusions, and the third is creation, which involves elements of existing knowledge. new so as to create new solutions or products (van der Zanden et al., 2020). At all levels of education there is one lesson to develop human thinking and train decision-making skills, namely mathematics.

Critical thinking is the ability to analyze, evaluate and solve problems objectively and systematically. This involves the ability to question, interpret and structure arguments logically, as well as the ability to use relevant evidence and information to support thinking and decision making. Critical thinking also includes the ability to identify assumptions, question the truth of claims, and recognize and overcome biases that may influence thinking. By thinking critically, a person can develop deeper understanding, more effective solutions, and better decision making in various contexts.

Facione's theory describes critical thinking as an analytical process that involves the ability to identify, interpret, evaluate, and integrate information critically. Facione's theory of critical thinking was developed by Dr. Peter A. Facione, an expert in the field of educational psychology. This theory summarizes various aspects and dimensions of critical thinking, and emphasizes the importance of this ability in rational decision making, problem solving, and assessing information (Facione , 1990).

Mathematics is a measuring tool for determining decisions on real problems and is related to mathematics so that mathematics is the basis for the thinking process to find solutions (Ati & Setawan, 2020). Mathematics can be learned by doing exercises to encourage thoughts in formulating problems, making plans to solve problems, preparing steps to solve problems, making guesses to see the solutions achieved (Kowiyah, 2012). Practicing logical and analytical thinking is needed to develop learning so that students can directly apply their understanding.

Mathematical ability refers to a person's ability to understand, apply, and master mathematical concepts in various contexts. Mathematical abilities cover various aspects, including: Understanding Mathematical Concepts, Application of Concepts, Problem Solving, Critical Thinking, Communication Ability, Mathematical Skills Development. Mathematical skills are very important in everyday life, as well as in various professional fields such as science, technology, engineering, finance, and many more. Good math skills help a person solve complex problems, make the right decisions, and think critically and creatively. Therefore, developing mathematical abilities is one of the main focuses in education throughout the world.

Number systems have existed since prehistoric times as evidenced by the introduction of the *tally system* used by ancient humans. The *tally* system is a

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number calculation system that is carried out using vertical lines as number symbols. The *tally* system has contributed to the development of mathematics as the initial foundation for the formation of real numbers. Real numbers are numbers that consist of rational and irrational numbers. Next, real numbers are added to imaginary units which are multiplied by a real number to form a complex number. Complex numbers are numbers that consist of real and imaginary numbers. Complex numbers can be applied to mathematical functions, namely polynomials.

A polynomial is a function that contains the addition of each term with a variable degree greater than zero whose coefficients are members of a ring set. A ring is a non-empty set that is equipped with addition and multiplication operations and satisfies several axioms. Complex numbers are a type of number that consists of a real part and an imaginary part. They are written in the form a + bi, where a is the real part, b is the imaginary part, and i is the imaginary unit, which is defined as the square root of -1. Complex numbers can be used to solve a variety of mathematics, physics, and engineering problems involving negative square roots, and they have wide applications in various fields of science. Complex number polynomials are a type of mathematical problem that involves operations with complex numbers and the application of algebraic concepts.

This type of research is descriptive qualitative research. This research was carried out in class XI, even semester 2024 at SMAN 1 Bojong, Pekalongan Regency.

The subjects in this research were 5 class XI students with high, medium and low mathematics abilities. Students were grouped based on mathematical ability, then the researcher chose one student from each mathematical ability group to be used as a research subject.

There are two instruments in this research, namely: main instrument (researcher) and supporting instruments (test sheet, interview guide and documentation). The data collection techniques used in this research were tests and interviews.

The data validity technique in this research uses time triangulation, namely by giving tests and interviews in different times or situations until certainty in the data is found so that it is suitable for analysis using data reduction techniques, data presentation and drawing conclusions.

Data analysis to analyze critical thinking processes based on *Facione's theory* in students with high, medium and low mathematical abilities in solving polynomial complex number problems.

#### DISCUSSION

The subjects in this research were 5 class XI students of SMA Negeri 1 Bojong. To be able to determine students' initial mathematical abilities, the determination is carried out through an initial ability test as an initial test that refers to learning indicators. The initial mathematics ability test of students in this research was carried out in April 2024. The test lasts for 60 minutes at the end of the lesson. The use of these hours is in accordance with the mathematics subject schedule . Based on the results of students' initial mathematics ability tests, the following data can be obtained.

Table 1. Description of Mathematics Ability of class XI students at SMA Negeri 1 Bojong.

No.	Mathematical Ability	Number of Students	Percentage
1.	Tall	1	20%
2.	currently	3	60%
3.	Low	1	20%
	Total	5	100%

The research results show that the average problem solving ability and mathematical critical thinking ability of students is at medium criteria. Based on the research results, it can be concluded that there is a significant difference in students' mathematical critical thinking abilities between the experimental group and the conventional group as a whole. These findings indicate that the teaching methods used in both groups have a significant effect on students' critical thinking skills (Susilo et al., 2020). Apart from that, there are differences in students' mathematical critical thinking abilities in the experimental class based on their initial mathematical abilities.

The research results show differences in the approaches and thinking processes of students with different mathematical abilities. Students with high mathematical abilities tend to use a more systematic and analytical approach in solving problems. They are able to identify patterns and apply the mathematical concepts they learn effectively. In contrast, students with lower mathematical abilities tend to rely on trial and error in solving problems. They may be less able to understand mathematical concepts in depth and tend to seek answers intuitively.

Critical thinking processes were also observed in this research. Students with higher math skills demonstrate the ability to identify patterns, solve problems, and provide justification for their steps. They are able to manage information well and carry out careful evaluations of the solutions they propose. On the other hand, students with lower mathematical abilities may have difficulty critically evaluating their solutions.

The research results also show that there is no interaction between the learning methods used and students' initial mathematical abilities on their critical mathematical abilities (Angraini et al., 2017). The findings of this research are consistent with previous research which shows the importance of problem solving and critical thinking skills in mathematics education (Susilo et al., 2020). Apart from that, it can be concluded that students' mathematical disposition plays a role in their critical thinking abilities. Based on the analysis of this research, it is clear that students' critical mathematical thinking skills play an important role in their ability to solve problems involving complex numbers and polynomials.

The analysis stage carried out by the researcher is making a list of questions for interviews, collecting data, and carrying out data analysis which is carried out by the researcher himself.

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# **Student Interview Results**

No	Question	Answer
1.	Did you experience difficulties	There are 3 opinions of students with
	when solving these questions?	High, Medium and Low ability:
		1. High ability students: Actually, I
		rarely experience significant
		difficulties in solving such
		problems. I feel comfortable
		with algebraic concepts and am
		familiar with complex number
		polynomials. I tend to remain
		calm when I am in a situation
		where I encounter a difficult
		problem and solve the problem
		systematically. If I have
		difficulty, I will revise relevant
		concepts or look for related
		examples to help me understand
		the problem. I also like to
		discuss things with friends or
		teachers if I'm really stuck.
		2. Medium ability students: Yes, of
		of the question that made me feel
		confused. Sometimes I need to
		repeat some steps or try a
		different approach to solve the
		problem. I also have to be careful
		not to make small mistakes that
		could change the final result.
		3. Low ability students: Yes ma'am, I
		often find it difficult. Polynomial
		problems are difficult, especially
		If there are complex numbers. I m
		start and many of the formulas
		are difficult to remember.
		especially time must determine
		the first step. I'm confused about
		where to start and what to do
		first. Then, when I try, the results
		are often different, so I'm
		confused about which one is
		right. But sometimes I'm
		emparrassed II I nave to ask for
		neip an uie unie, analu of being
		vourself first
		yoursen mist.

Table 2. Student interview results.

2.	Do you feel solving this problem	There are 3 opinions of students with
-	helped you develop critical	High. Medium and Low abilities:
	thinking skills?	1. High ability students: I feel that
	6	solving this problem helped me
		develop my critical thinking
		skills significantly. These
		questions require a deep
		understanding of complex
		mathematical concepts, as well
		as the ability to apply
		appropriate formulas and
		strategies. When I work on the
		problem, I must consider each
		step carefully and ensure that the
		solution I propose makes sense
		and is consistent with the
		information provided. I am also
		used to evaluating every solution
		I produce, to ensure that there
		are no errors or shortcomings in
		my reasoning.
		2. Medium ability student: Yes, I
		think solving this problem is very
		helpful in developing my critical
		thinking skills. When solving the
		problem, I had to consider the
		correct solution I also had to
		analyze the information provided
		in the problem, as well as
		evaluate the solution I proposed.
		This process forced me to think
		critically and logically. I feel my
		ability is quite good in identifying
		the problems given in the
		questions. I can quickly recognize
		the type of mathematical
		the information provided to
		understand what is required in
		the problem. I feel that I have
		used my critical thinking skills
		actively when solving this
		problem. The process of solving
		problems and evaluating
		solutions really helped me hone
		my critical thinking skills.
		3. Low ability student: Hmm,
		actually I think it's a bit difficult
		to say that solving this problem

	helped me develop critical
	thinking skills. I felt more like I
	was just trying out the steps and
	formulas that I remembered from
	previous lessons. I tried to find
	out what to do by trying out
	various formulas and strategies
	that the teacher had taught me.
	But, to be honest, I didn't really
	think deeply or evaluate my
	steps. There were several parts
	that made me a little confused,
	especially when looking for the
	next steps after getting stuck at
	one point. But I tried more and
	maybe asked a few friends.

Based on interviews with several students, it can be concluded that critical thinking skills play an important role in solving complex number polynomial problems. Students with high abilities tend to see problem solving as an opportunity to develop critical thinking skills by understanding each step carefully. Students of moderate ability also see it as an opportunity to think about steps more carefully, even though they may encounter difficulties. However, students with low abilities do not feel that problem solving helps in developing their critical thinking skills, perhaps because their focus is more on finding the right answer without understanding the process in depth . High-ability students demonstrate their ability to understand problems thoroughly, evaluate assumptions, overcome difficulties, and reflect on their thinking. This reflects the application of Facione's theory of critical thinking, which emphasizes the ability to identify, interpret, evaluate and integrate information critically. Thus, increasing critical thinking skills can support increasing abilities in solving complex mathematical problems such as complex number polynomials.

#### DISCUSSION

Based on the data in table 2, of the 5 class Next, to determine the research subjects, several students were selected from the 5 students with the criteria of being able to express their opinions well because the method used by researchers to collect data was by interviews based on the results of critical thinking ability tests carried out by the research subjects. Therefore, based on the suitability of the data obtained, the researcher determined research subjects with high, medium and low initial mathematical abilities as subjects to obtain main data related to information about the critical thinking abilities of students with high, medium and low mathematical abilities in working on complex number polynomial problems.

	Tuble 5. Subjects selected in cuen initial manemates usinty cutegory		
No	Early Mathematics skills	Test Scores	Subject Code
1.	Tall	70	S1
2.	Currently	90	S2
3.	Low	50	\$3

Table 3. subjects selected in each initial mathematics ability category

After selecting the subjects, the subjects were then given a critical thinking test and interviewed. The following is a description of students' critical thinking processes based on mathematical abilities:

# S1 Subjects:

On the Interpretation indicator, the subject writes down the information that is known and what is asked about from the question completely. It can be seen that the subject also identifies all the information known from the question. Figure 1 shows the results of subject S1's work on interpreting indicators.

```
Jika x3 - 4x2 + px+6 dan x2 +3x-2 dibagi (x-1) Memberikan
Sisa yang sama, nitai P adalah...
Penyelesaian
  x3-4x2+px+6 dibagi (x+1)
  Maka
   f(-1) = (-1)^3 - A(-1)^2 + P(-1) + 6
   f(-1) = -1 - 4 - P + 6
   f(-1) = 1 - P
   G(x) = x2 + 3x - 2 dibagi (x-1)
   Marca
                              f(-1) = G(-1)
    G(-1) = (-1)^2 + 3(-1) - 2
    G(-1) = 1 - 3 - 2
                              1-P = -4-1
     G(-1) = -4
                               -P = -5
                                 P
                                    = $
     Jadi nitai p adalah s.
```

Figure 1. The results of the S1 subject's work on interpreting indicators. T

he results of the subject's interview explaining polynomial material. The subject also explains the information known and the question intended in the problem correctly and clearly.

## Subject 2:

In the Interpretation indicator, the subject writes down the information that is known and what is asked about from the question completely. It can be seen that the subject also identifies all known information and understands the problem of the problem. Figure 2 shows the results of S2 subjects' work on interpreting indicators.

Jita X3-9X2+PX+6 dan X2+3X-2 dibagi (X+1) memberikan sisa yang sama nilai p adalah yelesaian - 9x2 + px +6 dibagi (x+1) maka  $f(-1) = (1)^{3} - 4(-1)^{2} + P(-1) + 6$ f(-1)G(x) = × 2+3×-2 dibagi (×+1) Maka 5(-1) = (-1) + 3(-1) - 2 f(-1) = G(-1)=-1-3-2 = - 6-1 + 1-6 = -7 P = 7 bdi nilai Padalah 7

Penyelesaian	
X3-4x2+8x+6 dibagi (x+1)	·> x3-4x3+ Px+6 frage (++1)
mara	=> $f(-1) = (-1)^3 - q(-1)^2 + q(-1) + 6$
$F(-1) = (-1)^{5} - 4(-1)^{7} + P(-1) + 6$	= -1-4-Pt6
f(-1) = -1 - 4 - P + 6	= 1-P
f(-1) = 1 - P	· ) x + 3x - 2 dilagi (x+1)
$B(x) = x^2 + 3x - 2$ dibagi (x+1)	=> q(-1) = (-1) + 3(-1) = 2
Mara:	> 4(-3-7
G(-1) = (-1) + 3(-1) - 2	
F(-1) = 1 - 3 - 2	NO
5(-1) = -4	(- ( nhm f (-1) = g (-1)
	1 - P = -4
f(-1) = f(-1)	-P: -4-1
1 - P = -4 - 1	- 9
-P =-6	
p = 5	p = 5
: nilai p adalah 5 //	

# Figures 2, 3, and 4. The results of the Master's subject's work on interpreting indicators

The results of the subject's interview explained that the questions were polynomial material. The subject also explains the known information and the question intended in the problem half correctly.

# Subject 3:

In indicator analysis, subjects write the information in the questions in the form of symbols and polynomial concepts. The results of the subject interview explain each symbol used and explain the steps used to solve the problem.

(Aika x<sup>3</sup>-4x<sup>3</sup>+px+6 Dan x<sup>2</sup>+3x-2 Bibagi (X+1) memberikan nisa Ya sama milai p-adalah... Penyelesaian : x3-4x2+px+6 846agi (x+1) malia :  $\frac{mall4}{f(-1)} = (-1)^3 - A(-1)^2 + p(-1) + 6$  f(-1) = -1 - 4 - p + 6f(-1) = 1 - p  $G(x) = x^{2} + 3x - 2$  Gibagi (x+1) maka:  $G(-1) = (-1)^{2} + 3(-1) - 2$ -> X+1 =0 X =- 1 G(-1) = t + 3 - 2G(-1) = 2f(-1) = G(-1) 1-p = 2 100 p = 2-1 P = 1

Figure 5. The results of the S3 subject's work on interpreting indicators.

The results of the subject's interview explaining polynomial material. The subject explains the known information and the question intended in the problem but the results of the calculation are incorrect.

#### CONCLUSION

Based on research conducted on 5 class XI students at SMA Negeri 1 Bojong, it is known that the initial mathematics ability test carried out in April 2024 showed quite promising results. The test is carried out for 60 minutes at the end of the lesson, according to the specified subject schedule. The results of this test are an initial picture of students' basic mathematical abilities before starting learning.

From table 1 which shows a description of students' mathematical abilities, it can be seen that the majority of students (60%) have moderate mathematical abilities. Meanwhile, 20% of students have high mathematical abilities and 20% of students have low mathematical abilities. Thus, it can be concluded that overall, class XI students at SMA Negeri 1 Bojong have good enough math skills to start learning. However, there are still some students who need more attention in improving their mathematics skills. Therefore, it is necessary to use appropriate strategies and approaches in the learning process so that all students can develop better mathematical skills.

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