IMPLEMENTATION OF GAUSS-SEIDEL ITERATION METHOD TO SOLVE COMPLEX LINEAR EQUATION SYSTEM

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ABSTRAK

Terdapat banyak metode untuk menyelesaikan sistem persamaan linear kompleks salah satunya yaitu menggunakan metode iterasi *Gauss-Seidel.* Ini merupakan metode penyelesaian persamaan serentak melalui proses iterasi dengan menggunakan nilai awal pada prosesnya, sehingga diperoleh nilai yang sesungguhnya dan syarat persamaan tersebut haruslah dominan secara diagonal. Tujuan penelitian ini dilakukan untuk mengeksplorasi metode iterasi *Gauss-Seidel* dalam menyelesaikan persamaan linear kompleks dengan empat persamaan empat variabel dan lima persamaan lima variabel. Penelitian ini merupakan penelitian kualitatif dengan fokus *Study Literature*. Langkah-langkah penyelesaian persamaan linear komplek menggunakan metode iterasi *Gauss-Seidel* yaitu diawali dengan penetapan sistem linear kompleks, mengubah persamaan menjadi bentuk matriks, selanjutnya mengintegrasikan matriks menjadi dominan secara diagonal sehingga mendapatkan solusi dari suatu sistem persamaan linear kompleks. Hasil penelitian menunjukkan bahwa metode iterasi *Gauss-Seidel* efektif dalam menyelesaikan empat persamaan empat variabel dan lima persamaan lima variabel pada sistem persamaan linear kompleks. Manfaat diadakannya penelitian ini yaitu untuk memberikan pemahaman kepada pembaca mengenai pengimplementasian metode *Gauss-Seidel* dalam menyelesaikan sistem persamaan linear kompleks.

Kata Kunci: Iterasi *Gauss-Seidel*, Bilangan Kompleks, Persamaan Linear Kompleks.

ABSTRACT

There are many methods to solve complex linear equation systems, one of which is using the Gauss-Seidel iteration method. This is a method of solving simultaneous equations through an iteration process using initial values in the process so as to obtain the true value and the condition that the equation must be diagonally dominant. The purpose of this study was conducted to explore the Gauss-Seidel iteration method in solving complex linear equations with four equations of four variables and five equations of five variables. This research is a qualitative research with Literature Study focus. The steps of solving complex linear equations using the Gauss-Seidel iteration method are starting with the determination of a complex linear system, converting the equation into matrix form, then integrating the matrix into diagonally dominant so as to get the solution of a system of complex linear equations. The results showed that the Gauss-Seidel iteration method is effective in solving four equations of four variables and five equations of five variables in the system of complex linear equations. The benefit of this research is to provide readers with an understanding of the implementation of the Gauss-Seidel method in solving complex linear equation systems.

Keywords: *Gauss-Seidel Iteration, Complex Numbers, Complex Linear Equations.*

INTRODUCTION

Complex linear equations are a form of mathematical equations that contain complex variables and are used in various fields of science such as physics, engineering, and mathematics. (Parida et al., 2024). In some cases, complex linear equations can be divided into several simpler linear equations and can be solved using numerical methods. One effective method for solving complex linear equations is the Gauss-Seidel iteration method.

The Gauss-Seidel iteration method is a numerical method used to solve complex systems of linear equations. The method works by dividing the system of equations into smaller parts and then solving each part separately. The result of each part is then used as input for the next part, so this process is repeated until the desired result is achieved. (Ihsan et al., 2024).

The scope of this research is the implementation of the Gauss-Seidel iteration method to solve complex linear equations. Previous research has been conducted by several researchers, such as those conducted by (Aryani et al., 2016) in the article "Gauss-Seidel and Generalized Gauss-Seidel Methods for Solving Complex Linear Equation Systems". They used the Gauss-Seidel iteration method to solve a system of complex linear equations where the SPL used must meet the Strictly Diagonally Dominant (SDD) condition, be symmetrical and positive definite.

The purpose of this study is to develop a more effective and efficient Gauss-Seidel iteration method for solving complex linear equations. In this study, we will use the Gauss-Seidel iteration method to solve a system of complex linear equations. We will also evaluate the efficiency and accuracy of the Gauss-Seidel method in solving complex linear equations.

In this research, we will use data from several sources, including scientific articles and textbooks. We will also use numerical software to implement the Gauss-Seidel iteration method and solve systems of complex linear equations.

DISCUSSION

Linear Equation System

A system of linear equations is a combination of two or more linear equations that are related to each other. Systems of linear equations play an important role in linear algebra. Linear algebra is often faced with the problem of finding the solution of a system of linear equations. The general form of a system of linear equations can be written as follows $Ax = y$ (Marzuki, 2015).

A system of linear equations is a set of linear equations consisting of m linear equations, $L_{1,} L_{2,} \dots L_m$, with n unknown variables $x_{1,} x_{2,} \dots x_m$ which are arranged in the following form:

$$a_{11}\, x_1 + a_{12}\, x_2 + \cdots + a_{1n}\, x_n = b_1$$
$$a_{21}\, x_1 + a_{22}\, x_2 + \cdots + a_{2n}\, x_n = b_2 \qquad (1.1)$$
$$a_{m1}\, x_1 + a_{m2}\, x_2 + \cdots + a_{mn}\, x_n = b_m$$

with a_{ij} is the coefficient of the unknown variable x_j in equation liand the number l_i,and the number b_i is the constant of the equation li.

The system of linear equations in equation (1.1) consisting of m linear equations with n variables is equivalent to the matrix equation $AX = B$, namely:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

with $A = [a_{ij}]$ is the coefficient matrix, $X = [x_j]$ is a column vector of unknown variables, and $B = [b_i]$ is a column vector of constants.

Complex Linear Equation System

A system of complex linear equations is a system of linear equations whose coefficients are complex numbers. In the system of complex linear equations, there are complex numbers. These complex numbers can be written as follows:

$$C_{ij} = p_{ij} + iq_{ij}$$

$$Z_i = x_i + iy_i$$

$$W_i = u_i + iv_i$$

The first step taken to find the solution of the complex linear equation system is to convert the coefficient matrix A which is of size $n \times n$ into a matrix of size $2n \times 2n$.. To convert matrix A into a matrix of size $2n \times 2n$., then the following equation is used:

$$\Sigma_{j=1}^{n} \quad C_{ij} \quad Z_i \quad = \quad W_I \qquad (1.2)$$

The above equation can be trenslated into:

$$\Sigma_{j=1}^{n} \quad p_{ij} + iq_{ij} \quad x_i + iy_i \ = \ u_i + iv_i \qquad (1.3)$$

with

$V = v_i$

$U = u_i$

$P = p_{ij}$

$Q = q_{ij}$

$X = x_i$

$Y = y_i$ fpr $i, j = 1, 2, \ldots, n$ (1.4)

Based on equation (1.3) we get a new matrix of size $2n \times 2n$ an follows:

$$\begin{matrix} P & -Q \\ Q & P \end{matrix} \quad \begin{matrix} X \\ Y \end{matrix} = \begin{matrix} U \\ V \end{matrix} \qquad (1.5)$$

Gauss-Seidel Iteration Method

Gauss-Seidel iteration is a method of solving simultaneous equations through an iteration process. The Gauss-Seidel iteration method is a repeated recursion process to approximate an unknown number. (X). As a starting point in the recursion process, the initial value is required and usually is x = 0. In the next process, the value that is known at the previous stage x_1 is used to find the value at the next stage x_2. The process continues to repeat until the true value is obtained or stops if a certain error tolerance has been reached. x or stops if a certain error tolerance has been reached.

The following set of linear equations:

$$a_{11}\, x_1 + a_{12}\, x_2 + \cdots + a_{1n}\, x_n = b_1$$
$$a_{21}\, x_1 + a_{22}\, x_2 + \cdots + a_{2n}\, x_n = b_2$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$a_{m1}\, x_1 + a_{m2}\, x_2 + \cdots + a_{mn}\, x_n = b_m$$

The i-th equation of the above equation is $a_{11}\, x_1 + a_{12}\, x_2 + \cdots + a_{1n}\, x_n = b_n$ with $i = 1,2,3, \ldots , k$.

Thus the Gauss-Seidel method is expressed as:

$$x_i{}^{(k)} = \frac{1}{aii}\left[- \sum_{j=1}^{i-1}\left(a_{ij}x_j{}^{(k)}\right) - \sum_{j=i+1}^{n}\left(a_{ij}x_j{}^{(k-1)}\right) + b_i\right]$$

$untuk\ i = 1, 2, 3, \ldots, n\ dan\ a_{ii} \neq 0.$

Based on (3), then the general formula of the gauss-seidel iteration method becomes:

$$x_s{}^{(k)} = \frac{1}{p_{ss}}\left[- \sum_{t=1}^{s-1}\left(p_{st}x_t{}^{(k)}\right) - \sum_{t=s+1}^{n}\left(p_{st}x_t{}^{(k-1)}\right) - \sum_{t=1+1}^{n}\left(-q_{st}y_t{}^{(k-1)}\right) + u_s\right]$$

and (1.6)

$$x_s{}^{(k)} = \frac{1}{p_{ss}}\left[- \sum_{t=1}^{n}\left(p_{st}x_t{}^{(k)}\right) - \sum_{t=s+1}^{n}\left(p_{st}x_t{}^{(k-1)}\right) - \sum_{t=1}^{n}\left(q_{st}y_t{}^{(k)}\right) + v_s\right]$$

$for\ s = 1, 2, \ldots , n;\ p_{ss} \neq 0\ and\ k = 1, 2, \ldots$

To solve the system of equations with the Gauss-Seidel method, an initial approximation value is required, namely x_0, usually unknown and we choose $x_0 = 0$. Therefore, a sufficient condition for the Gauss-Seidel method to converge is:

$$\left|a_{ij}\right| > \sum_{j=1}^{n} a_{ij}, i = 1,2,3, \ldots, n \qquad (1.7)$$

In other words, the Gauss-Seidel method will converge if the coefficients of the diagonally dominant matrix

Example 1.1

Solve the following system of linear equations using Gauss-Seidel iteration, with variable initial point values, namely $x_1^0 = x_2^0 = x_3^0 = x_4^0 = x_5^0 = 0$.

$$8x_1 + x_2 + 2x_3 + 3x_4 + x_5 = 20$$
$$4x_1 + 9x_2 + x_3 + x_4 + x_5 = 50$$
$$3x_1 + x_2 + 7x_3 + x_4 + x_5 = 32$$
$$x_1 + x_2 + 7x_3 + 8x_4 + 2x_5 = 40$$

$$x_1 + 2x_2 + 3x_3 + 2x_4 + 9x_5 = 45$$

Completion:

Based on the system of linear equations obtained matrix A which is:

$$A = \begin{matrix} 8 & 1 & 2 & 3 & 1 \\ 4 & 9 & 1 & 1 & 1 \\ 3 & 1 & 7 & 1 & 1 \\ 3 & 1 & 1 & 8 & 2 \\ 1 & 2 & 3 & 2 & 9 \end{matrix}$$

With variable matrix x is:

$$x = \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix}$$

The constant matrix b is:

$$b = \begin{matrix} 20 \\ 50 \\ 32 \\ 40 \\ 45 \end{matrix}$$

So, $AX = B$

$$A = \begin{matrix} 8 & 1 & 2 & 3 & 1 \\ 4 & 9 & 1 & 1 & 1 \\ 3 & 1 & 7 & 1 & 1 \\ 3 & 1 & 1 & 8 & 2 \\ 1 & 2 & 3 & 2 & 9 \end{matrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} = \begin{matrix} 20 \\ 50 \\ 32 \\ 32 \\ 45 \end{matrix}$$

Finally, we iterate using equation (1.6) with the condition that it must fulfill equation (1.7), we will get the value:

Iteration:

$$x_1^k = \frac{1}{8}\left(20 - x_2^{(k-1)} - 2x_3^{(k-1)} - 3x_4^{(k-1)} - x_5^{(k-1)}\right)$$

$$x_2^k = \frac{1}{9}\left(50 - 4x_1^k - x_3^{(k-1)} - x_4^{(k-1)} - x_5^{(k-1)}\right)$$

$$x_3^k = \frac{1}{7}\left(32 - 3x_1^k - x_2^k - x_4^{(k-1)} - x_5^{(k-1)}\right)$$

$$x_4^k = \frac{1}{8}\left(40 - 3x_1^k - x_2^k - x_3^k - 2x_5^{(k-1)}\right)$$

$$x_5^k = \frac{1}{9}(45 - x_1^k - 2x_2^k - 3x_3^k - 2x_4^k)$$

The next iteration can be seen in table 1.1 below

Table 1.1 Iteration Results Example 1.1

Iterasi	x_1	x_2	x_3	x_4	x_5
	0	0	0	0	0
1.	2.5000	4.4444	2.8651	3.7738	1.9409
2.	-0.4296	4.7932	3.2544	3.5625	2.1061
3.	-0.5120	4.7917	3.2965	3.5265	2.1096
4.	-0.5092	4.7894	3.3003	3.5250	2.1088
5.	-0.5092	4.7892	3.3007	3.5252	2.1087
6.	-0.5094	4.7892	3.3007	3.5253	2.1087
7.	-0.5094	4.7892	3.3007	3.5253	2.1087

Based on the iteration table above, the solution of the system of linear equations is: $x_1 = -0.5094, x_2 = 4.7892,$ and $x_3 = 3.3007, x_4 = 3.5253, x_5 = 2.1087.$ Because if it continues until the nth iteration, the result will be the same as the previous iteration.

This research is qualitative research with a focus on literature study. The research method that the author uses is the literature study method with the following steps:

1) First know the system of complex linear equations
2) Transforming the equation into matrix form A which is of size $n \times n$ as in equation (1.3)
3) Transforming the matrix A which is of size $n \times n$ as in equation (1.3) into a matrix with the entries $P = P_{ij}$, $Q = q_{ij}$, $U = u_{ij}$, $V = v_{ij}$, $X = x_{ij}$, $Y = y_{ij}$ as in equation (1.4)
4) Next change the matrix A which is of size $n \times n$ into the form of a matrix C sized $2n \times 2n$ with equation (1.5)
5) Determining the matrix C whose size $2n \times 2n$ is diagonally dominant with equation (1.7)
6) Iterate the matrix C that is diagonally dominant.

7) Obtain the solution of a system of complex linear equations.

Complex Linear Equations with 4 Equations and 4 Variables

Given a system of complex linear equations with 4 equations and 4 variables as follows:

$$(20+3i)x_1 + (1-2i)x_2 + (3-i)x_3 + (4+3i)x_4 = 10-3i$$
$$(1-2i)x_1 + (18+2i)x_2 + (3+2i)x_3 + (5-i)x_4 = 8-2i$$
$$(2+2i)x_1 + (2+i)x_2 + (16+i)x_3 + (3-3i)x_4 = 7-2i$$
$$(2-i)x_1 + (2-2i)x_2 + (2+2i)x_3 + (15+2i)x_4 = 12-3i$$

Solve the system of complex linear equations above using the Gauss Seidel iteration method with the initial point value of the variable, namely $x_1^0 = x_2^0 = x_3^0 = x_4^0 = y_1^0 = y_2^0 = y_3^0 = y_4^0$ with $\varepsilon = 10^{-4}$.

Solution:

The first step is to convert the system of complex linear equations into matrix equation form AX=B into:

$$\begin{bmatrix} 20+3i & 1-2i & 3-i & 4+3i \\ 1-2i & 18+2i & 3+2i & 5-i \\ 2+2i & 2+i & 16+i & 3-3i \\ 2-i & 2-2i & 2+2i & 15+2i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10-3i \\ 8-2i \\ 7-2i \\ 12-3i \end{bmatrix}$$

The second step is to transform the matrix AX=B into equation (1.5) as follows:

$$P = \begin{bmatrix} 20 & 1 & 3 & 4 \\ 1 & 18 & 3 & 5 \\ 2 & 2 & 16 & 3 \\ 2 & 2 & 2 & 15 \end{bmatrix}, \quad Q = \begin{bmatrix} 3 & -2 & -1 & 3 \\ -2 & 2 & 2 & -1 \\ 2 & 1 & 1 & -3 \\ -1 & -2 & 2 & 2 \end{bmatrix}, \quad U = \begin{bmatrix} 10 \\ 8 \\ 7 \\ 12 \end{bmatrix}$$

$$V = \begin{bmatrix} -3 \\ -2 \\ -2 \\ -3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Thus, it can be converted into the form $\begin{bmatrix} P & -Q \\ Q & P \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} U \\ V \end{bmatrix}$ as follows:

$$\begin{bmatrix} 20 & 1 & 3 & 4 & -3 & 2 & 1 & -3 \\ 1 & 18 & 3 & 5 & 2 & -2 & -2 & 1 \\ 2 & 2 & 16 & 3 & -2 & -1 & -1 & 3 \\ 2 & 2 & 2 & 15 & 1 & 2 & -2 & -2 \\ 3 & -2 & -1 & 3 & 20 & 1 & 3 & 4 \\ -2 & 2 & 2 & -1 & 1 & 18 & 3 & 5 \\ 2 & 1 & 1 & -3 & 2 & 2 & 16 & 3 \\ -1 & -2 & 2 & 2 & 2 & 2 & 2 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \\ 12 \\ -3 \\ -2 \\ -2 \\ -3 \end{bmatrix}$$

The next step is to ensure the matrix $\begin{bmatrix} P & -Q \\ Q & P \end{bmatrix}$ is the diagonally dominant matrix.

$\begin{bmatrix} P & -Q \\ Q & P \end{bmatrix}$ is the diagonally dominant matrix, because

$$|20| > |1| + |3| + |4| + |-3| + |2| + |1| + |-3|$$
$$|18| > |1| + |3| + |5| + |2| + |-2| + |-2| + |1|$$
$$|16| > |2| + |2| + |3| + |-2| + |-1| + |-1| + |3|$$
$$|15| > |2| + |2| + |2| + |1| + |2| + |-2| + |-2|$$
$$|20| > |3| + |-2| + |-1| + |3| + |1| + |3| + |4|$$
$$|18| > |-2| + |2| + |2| + |-1| + |1| + |3| + |5|$$
$$|16| > |2| + |1| + |1| + |-3| + |2| + |2| + |3|$$
$$|15| > |-1| + |-2| + |2| + |2| + |2| + |2| + |2|$$

Next, we iterate the new matrix based on equation (1.6) to obtain:

$$x_1^k = \frac{1}{20}\Big(10 - x_2^{(k-1)} - 3x_3^{(k-1)} - 4x_4^{(k-1)} + 3y_1^{(k-1)} - 2y_2^{(k-1)} - y_3^{(k-1)} + 3y_4^{(k-1)}\Big)$$

$$x_2^k = \frac{1}{18}\Big(8 - x_1^{(k)} - 3x_3^{(k-1)} - 5x_4^{(k-1)} - 2y_1^{(k-1)} + 2y_2^{(k-1)} + 2y_3^{(k-1)} - y_4^{(k-1)}\Big)$$

$$x_3^k = \frac{1}{16}\Big(7 - 2x_1^{(k)} - 2x_2^{(k)} - 3x_4^{(k-1)} + 2y_1^{(k-1)} + y_2^{(k-1)} + y_3^{(k-1)} - 3y_4^{(k-1)}\Big)$$

$$x_4^k = \frac{1}{15}\Big(12 - 2x_1^{(k)} - 2x_2^{(k)} - 2x_3^{(k)} - y_1^{(k-1)} - 2y_2^{(k-1)} + 2y_3^{(k-1)} + 2y_4^{(k-1)}\Big)$$

$$y_1^k = \frac{1}{20}\Big(-3 - 3x_1^{(k)} + 2x_2^{(k)} + x_3^{(k)} - 3x_4^{(k)} - y_2^{(k-1)} - 3y_3^{(k-1)} - 4y_4^{(k-1)}\Big)$$

$$y_2^k = \frac{1}{18}\Big(-2 + 2x_1^{(k)} - 2x_2^{(k)} - 2x_3^{(k)} + x_4^{(k)} - y_1^{(k)} - 3y_3^{(k-1)} - 5y_4^{(k-1)}\Big)$$

$$y_3^k = \frac{1}{16}\Big(-2 - 2x_1^{(k)} - x_2^{(k)} - x_3^{(k)} + 3x_4^{(k)} - 2y_1^{(k)} - 2y_2^{(k-1)} - 3y_4^{(k-1)}\Big)$$

$$y_4^k = \frac{1}{15}\Big(-3 + x_1^{(k)} + 2x_2^{(k)} - 2x_3^{(k)} - 2x_4^{(k)} - 2y_1^{(k)} - 2y_2^{(k-1)} - 2y_3^{(k-1)}\Big)$$

where k is the number of iterations.

The iteration process using this method obtained the results as can be seen in Appendix 1.

Based on Appendix 2, the iteration process stops at the 6th iteration by entering the value of $\varepsilon = 10^{-4}$. So the complex linear equation system is:

$x_1 = 0.2436, x_2 = 0.2312, x_3 = 0.2711,\ x_4 = 0.6859,$

$y_1 = -0.2037i, y_2 = -0.0232i, y_3 = 0.0175i, y_4 = -0.2526i.$

So the z-value solution for the given complex linear system of equations based on the x and y values obtained then:

$z_{1=}\ 0.2436\ -\ 0.2037i$

$z_{2=}\ 0.2312\ -\ 0.0232i$

$z_{3=}\ 0.2711\ +\ 0.0175i$

$z_{4=}\ 0.6859\ -\ 0.2526i$

Complex Linear Equations with 5 Equations and 5 Variables

Given a system of complex linear equations with 5 equations and 5 variables as follows:

$$(30 + 8i)x_1 + (2 - 6i)x_2 + (3 - 2i)x_3 + (4 + 2i)x_4 + (2 - i)x_5 = 25 + 3i$$
$$(2 - i)x_1 + (25 + 7i)x_2 + (2 - 2i)x_3 + (5 - 2i)x_4 + (3 + 5i)x_5 = 29 - 3i$$
$$(3 + 3i)x_1 + (5 + 3i)x_2 + (45 + 6i)x_3 + (6 - i)x_4 + (5 + 2i)x_5 = 23 - 2i$$
$$(7 + 2i)x_1 + (3 - 2i)x_2 + (5 + i)x_3 + (50 + 5i)x_4 + (6 - 3i)x_5 = 36 - 3i$$
$$(6 - 3i)x_1 + (8 + 2i)x_2 + (2 + 2i)x_3 + (7 + 5i)x_4 + (40 + 4i)x_5 = 30 - 2i$$

Solve the above system of complex linear equations using the Gauss Seidel iteration method with variable starting point values, namely:

$x_1^0 = x_2^0 = x_3^0 = x_4^0 = x_5^0 = y_1^0 = y_2^0 = y_3^0 = y_4^0 = y_5^0\ \varepsilon = 10^{-5}$

Solution:

The first step is to convert the system of complex linear equations into matrix equation form AX=B into:

$$\begin{bmatrix} 30+8i & 2-6i & 3-2i & 4+2i & 2-i \\ 2-i & 25+7i & 2-2i & 5-2i & 3+5i \\ 3+3i & 5+3i & 45+6i & 6-i & 5+2i \\ 7+2i & 3-2i & 5+i & 50+5i & 6-3i \\ 6-3i & 8+2i & 2+2i & 7+5i & 40+4i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 25+3i \\ 29-3i \\ 23-2i \\ 36-3i \\ 30-2i \end{bmatrix}$$

The second step is to transform the matrix AX=B into equation (1.5) as follows:

$$P = \begin{bmatrix} 30 & 2 & 3 & 4 & 2 \\ 2 & 25 & 2 & 5 & 3 \\ 3 & 5 & 45 & 6 & 5 \\ 7 & 3 & 5 & 50 & 6 \\ 6 & 8 & 2 & 7 & 40 \end{bmatrix}, Q = \begin{bmatrix} 8 & -6 & -2 & 2 & -1 \\ -1 & 7 & -2 & -2 & 5 \\ 3 & 3 & 6 & -1 & 2 \\ 2 & -2 & 1 & 5 & -3 \\ -3 & 2 & 2 & 5 & 4 \end{bmatrix},, \quad U = \begin{bmatrix} 25 \\ 29 \\ 23 \\ 36 \\ 30 \end{bmatrix}$$

$$V = \begin{bmatrix} 3 \\ -3 \\ -2 \\ -3 \\ -2 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$$

Thus, it can be converted into the form $\begin{bmatrix} P & -Q \\ Q & P \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} U \\ V \end{bmatrix}$ as follows:

$$\begin{bmatrix} 30 & 2 & 3 & 4 & 2 & -8 & 6 & 2 & -2 & 1 \\ 2 & 25 & 2 & 5 & 3 & 1 & -7 & 2 & 2 & -5 \\ 3 & 5 & 45 & 6 & 5 & -3 & -3 & -6 & 1 & -2 \\ 7 & 3 & 5 & 50 & 6 & -2 & 2 & -1 & -5 & 3 \\ 6 & 8 & 2 & 7 & 40 & 3 & -2 & -2 & -5 & -4 \\ 8 & -6 & -2 & 2 & -1 & 30 & 2 & 3 & 4 & 2 \\ -1 & 7 & -2 & -2 & 5 & 2 & 25 & 2 & 5 & 3 \\ 3 & 3 & 6 & -1 & 2 & 3 & 5 & 45 & 6 & 5 \\ 2 & -2 & 1 & 5 & -3 & 7 & 3 & 5 & 50 & 6 \\ -3 & 2 & 2 & 5 & 4 & 6 & 8 & 2 & 7 & 40 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 25 \\ 29 \\ 23 \\ 36 \\ 30 \\ 3 \\ -3 \\ -2 \\ -3 \\ -2 \end{bmatrix}$$

The next step is to ensure the matrix $\begin{bmatrix} P & -Q \\ Q & P \end{bmatrix}$ is the diagonally dominant matrix.

$\begin{bmatrix} P & -Q \\ Q & P \end{bmatrix}$ is the diagonally dominant matrix, because

$$|30| > |2| + |3| + |4| + |2| + |-8| + |6| + |2| + |-2| + |1|$$

$$|25| > |2| + |2| + |5| + |3| + |1| + |-7| + |2| + |2| + |-5|$$

$$|45| > |3| + |5| + |6| + |5| + |-3| + |-3| + |-6| + |1| + |-2|$$

$$|50| > |7| + |3| + |5| + |6| + |-2| + |2| + |-1| + |-5| + |3|$$

$$|40| > |6| + |8| + |2| + |7| + |3| + |-2| + |-2| + |-5| + |-4|$$

$$|30| > |8| + |-6| + |-2| + |2| + |-1| + |2| + |3| + |4| + |2|$$

$$|25| > |-1| + |7| + |-2| + |-2| + |5| + |2| + |2| + |5| + |3|$$

$$|45| > |3| + |3| + |6| + |-1| + |2| + |3| + |5| + |6| + |5|$$

$$|50| > |2| + |-2| + |1| + |5| + |-3| + |7| + |3| + |5| + |6|$$

$$|40| > |-3| + |2| + |2| + |5| + |4| + |6| + |8| + |2| + |7|$$

Next, we iterate the new matrix based on equation (1.6) to obtain:

$$x_1^k = \frac{1}{30}\Big(25 - 2x_2^{(k-1)} - 3x_3^{(k-1)} - 4x_4^{(k-1)} - 2x_5^{(k-1)} + 8y_1^{(k-1)} - 6y_2^{(k-1)} - 2y_3^{(k-1)} + 2y_4^{(k-1)} - y_5^{(k-1)}\Big)$$

$$x_2^k = \frac{1}{25}\Big(29 - 2x_1^{(k)} - 2x_3^{(k-1)} - 5x_4^{(k-1)} - 3x_5^{(k-1)} - y_1^{(k-1)} + 7y_2^{(k-1)} - 2y_3^{(k-1)} - 2y_4^{(k-1)} + 5y_5^{(k-1)}\Big)$$

$$x_3^k = \frac{1}{45}\Big(23 - 3x_1^{(k)} - 5x_2^{(k)} - 6x_4^{(k-1)} - 5x_5^{(k-1)} + 3y_1^{(k-1)} + 3y_2^{(k-1)} + 6y_3^{(k-1)} - y_4^{(k-1)} + 2y_5^{(k-1)}\Big)$$

$$x_4^k = \frac{1}{50}\Big(36 - 7x_1^{(k)} - 3x_2^{(k)} - 5x_3^{(k)} - 6x_5^{(k-1)} + 2y_1^{(k-1)} - 2y_2^{(k-1)} + y_3^{(k-1)} + 5y_4^{(k-1)} - 3y_5^{(k-1)}\Big)$$

$$x_5^k = \frac{1}{40}\Big(30 - 6x_1^{(k)} - 8x_2^{(k)} - 2x_3^{(k)} - 7x_4^{(k)} - 3y_1^{(k-1)} + 2y_2^{(k-1)} + 2y_3^{(k-1)} + 5y_4^{(k-1)} + 4y_5^{(k-1)}\Big)$$

$$y_1^k = \frac{1}{30}\Big(3 - 8x_1^{(k)} + 6x_2^{(k)} + 2x_3^{(k)} - 2x_4^{(k)} + x_5^{(k)} - 2y_2^{(k-1)} - 3y_3^{(k-1)} - 4y_4^{(k-1)} - 2y_5^{(k-1)}\Big)$$

$$y_2^k = \frac{1}{25}\Big(-3 + x_1^{(k)} - 7x_2^{(k)} + 2x_3^{(k)} + 2x_4^{(k)} - 5x_5^{(k)} - 2y_1^{(k)} - 2y_3^{(k-1)} - 5y_4^{(k-1)} - 3y_5^{(k-1)}\Big)$$

$$y_3^k = \frac{1}{45}\Big(-2 - 3x_1^{(k)} - 3x_2^{(k)} - 6x_3^{(k)} + x_4^{(k)} - 2x_5^{(k)} - 3y_1^{(k)} - 5y_2^{(k)} - 6y_4^{(k-1)} - 5y_5^{(k-1)}\Big)$$

$$y_4^k = \frac{1}{50}\Big(-3 - 2x_1^{(k)} + 2x_2^{(k)} - x_3^{(k)} - 5x_4^{(k)} + 3x_5^{(k)} - 7y_1^{(k)} - 3y_2^{(k)} - 5y_3^{(k)} - 6y_5^{(k-1)}\Big)$$

$$y_5^k = \frac{1}{40}\Big(-2 + 3x_1^{(k)} - 2x_2^{(k)} - 2x_3^{(k)} - 5x_4^{(k)} - 4x_5^{(k)} - 6y_1^{(k)} - 8y_2^{(k)} - 2y_3^{(k)} - 7y_4^{(k)}\Big)$$

where k is the number of iterations.

The iteration process using this method obtained the results as can be seen in Appendix 2.

Based on Appendix 2, the iteration process stops at the 6th iteration by entering the value of $\varepsilon = 10^{-5}$. So the system of complex linear equations is:

$x_1 = 0.7691$, $x_2 = 0.9373$, $x_3 = 0.1150$, $x_4 = 0.2507$, $x_5 = 0.4016$,
$y_1 = 0.0984i$, $y_2 = -0.1752i$, $y_3 = -0.0697i$,$y_4 = 0.0561i$,$y_5 - 0.0088i$
So the z-value solution for the given complex linear system of equations based on the x and y values obtained then:

$z_{1=}\ 0.7691 + 0.0984i$

$z_{2=}\ 0.9373 - 0.1752i$

$z_{3=}\ 0.1150 - 0.0697i$

$z_{4=}\ 0.2507 + 0.0561i$

$z_{5=}\ 0.4016 - 0.0088i$

CONCLUSION

This research discusses the Gauss-Seidel iteration method in solvingcomplex linear equation systems with four equations of four variables and five equations of five variables. The results show that the Gauss-Seidel iteration method is effective in solving the system of equations. With the Gauss-Seidel iteration method, complex linear equations can be converted into matrix form and the solution can be found. Iteration is performed until it reaches a convergent condition. The application of the method is based on the literature study and the context of the research that has been done qualitatively.

The results show that the solution of the given system of linear equations can be found by the Gauss-Seidel iteration method. This method is effective and efficient in solving complex equations with specified initial values. This study provides a clear picture of the Gauss-Seidel method in solving complex linear equation systems. Thus, this method can be used to solve complex linear equation systems quite well. The Gauss-Seidel method provides accurate results in solving complex equations, so it can be widely used in various fields of science that require solving complex linear equations. The method can help in analysing and solving systems of complex linear equations in real contexts in various fields of science.

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